Written Reexam at the Department of Economics summer 2020

Economics of the Environment and Climate Change

Final Reexam

August 20, 2020

(3-hour open book exam)

Answers only in English.

This exam question consists of 5 pages in total, including this front page.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

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You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

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Written Reexam in the Economics of the Environment and Climate Change, Spring 2020

OPTIMAL ENVIRONMENTAL TAX POLICY

In the following you will be asked to study the optimal environmental tax policy in an economy where the representative consumer consumes a "clean" (non-polluting) good x_1 and a "dirty" (polluting) good x_2. The consumer also gets utility from a public good *G* provided by the government, whereas the consumer's labour supply *L* and the level of pollution *P* generate disutility. The utility function *U* of the representative consumer takes the following form where η and ε are constant parameters:

$$U = \left(\frac{\eta}{\eta-1}\right) x_1^{\frac{\eta-1}{\eta}} + \left(\frac{\varepsilon}{\varepsilon-1}\right) x_2^{\frac{\varepsilon-1}{\varepsilon}} + g(G) - L - e(P), \tag{1}$$

$$\eta > 1, \quad \varepsilon > 1, \quad g'(G) > 0, \quad g''(G) < 0, \quad e'(P) > 0, \quad e''(P) > 0.$$

The term $-L$ in (1) measures the disutility from work. The function $g(G)$ measures the utility from the public good, and the function $e(P)$ captures the disutility from pollution. The assumptions $g'(G) > 0$ and $g''(G) < 0$ mean that the marginal utility of the public good is positive but decreasing, while the assumptions $e'(P) > 0$ and $e''(P) > 0$ reflect that the marginal disutility from pollution is positive and increasing.

The production and/or consumption of one unit of the dirty good x_2 causes emission of one unit of a pollutant. All consumers in the economy are identical, and their total number is *n*, so the total level of pollution is

$$P = nx_2. \tag{2}$$

The number of consumers *n* is very large, so the individual consumer feels unable to influence the total level of pollution *P*. Thus the individual consumer takes *P* as given when making her decisions

on consumption and labour supply. All consumers feel the same disutility from pollution, so the pollutant considered is "uniformly mixing".

The production of all the goods x_1, x_2 and *G* takes place under constant returns to scale with labour as the only input. Without any loss of generality, we can choose our unit for measuring *L* such that the wage rate for one unit of labour is 1. We can also choose the units for measuring the quantities x_1, x_2 and *G* such that it takes exactly one unit of labour to produce one unit of each good. When the wage rate is 1, the constant marginal cost of producing each good will then also be 1, so under perfect competition the equilibrium prices of each good will likewise be 1. We can therefore write the consumer's budget constraint in the following simple form where *t* is a pollution tax and *T* is a lump-sum tax levied by the government:

$$x_1 + (1+t)x_2 = L - T. \tag{3}$$

The left-hand side of (3) is total consumer spending on private goods, and the right-hand side is the consumer's disposable income, since *L* is total labour income, given that the wage rate is 1.

The representative consumer wants to maximise her utility (1) subject to the budget constraint (3). The Lagrange function ℓ^c corresponding to this problem is

$$\ell^c = \left(\frac{\eta}{\eta-1}\right) x_1^{\frac{\eta-1}{\eta}} + \left(\frac{\varepsilon}{\varepsilon-1}\right) x_2^{\frac{\varepsilon-1}{\varepsilon}} + g(G) - L - e(P) - \lambda\left[x_1 + (1+t)x_2 - L + T\right], \tag{4}$$

where λ is the Lagrange multiplier associated with the consumer's budget constraint, and where the consumer takes *P* as well as the government's policy instruments *G*, *t* and *T* as given.

Question 1. Show that the first-order conditions for the solution to the consumer's utility maximisation problem combined with the consumer budget constraint imply the following demand functions for the two private goods,

$$x_1 = 1, \qquad x_2 = (1+t)^{-\varepsilon}, \tag{5}$$

and the following labour supply function,

$$L = 1 + (1+t)^{1-\varepsilon} + T. \tag{6}$$

Give a brief economic explanation for the way the tax instruments *t* and *T* influence labour supply (Hint: Recall the assumption in (1) that $\varepsilon > 1$). (End of Question 1).

When the consumer's optimum conditions (5) and (6) are plugged into the direct utility function (1) along with (2), we obtain the *indirect* utility function which expresses the consumer's maximum attainable level of utility, given the government's choice of the policy instruments *t*, *T* and *G*.

Question 2. Show by using (1), (2), (5) and (6) that the consumer's indirect utility function *V* can be written as

$$V = \frac{1}{\eta - 1} + \frac{(1+t)^{1-\varepsilon}}{\varepsilon - 1} - T + g(G) - e\left(n(1+t)^{-\varepsilon}\right). \tag{7}$$

Explain briefly the ways in which the pollution tax *t* affects the consumer's maximum attainable utility level. (End of Question 2).

The government finances its provision of the public good *G* by the revenues from the pollution tax and from the lump sum tax *T* levied on all consumers. Since all consumers are identical and the price (unit cost) of the public good is 1, the government's budget constraint is

$$n(tx_2 + T) = G \quad \Leftrightarrow \quad T = \frac{G}{n} - tx_2. \tag{8}$$

Question 3. Insert the consumer's optimal demand for x_2 in (8) and use the resulting expression to eliminate *T* from (7) so that the consumer's indirect utility is expressed solely as a function of the two policy variables *t* and *G*.

Question 4. The government chooses its policy instruments *G* and *t* with the purpose of maximising the utility of the representative consumer while obeying the government budget constraint. Use the indirect utility function derived in Question 3 to derive the first-order conditions for the government's optimal choice of *G* and *t*. Explain the economic intuition behind the first-order condition for the optimal choice of *G*. Rewrite the first-order condition for the optimal value of *t* to isolate *t* on the left-hand side of the equation so you get a simple expression for the optimal pollution tax. Explain the economic intuition behind this expression. (Hint: When stating your final expression for the optimal value of *t*, you may use the fact that $e'\left(n\left(1+t\right)^{-\varepsilon}\right)=e'\left(P\right)$.)

Question 5. Now suppose that the economy actually consists of many heterogeneous consumers with different earnings capabilities resulting in an unequal distribution of income. Suppose further that the government cannot impose individualized non-distortionary lump sum taxes to correct for undesirable inequalities in income distribution but that it can impose a progressive income tax. Discuss whether a government in such a society would want to stick to the rule for the optimal pollution tax that you derived in Question 4. (Hint: You are not asked to do any formal mathematical analysis here; a verbal discussion suffices).

Question 6. Environmental economists usually assume that the objective of pollution tax policy is to achieve economic efficiency in resource allocation. Discuss briefly some other objectives that might also be legitimate and relevant targets for policies against pollution.